Find the absolute extrema of $f(x)=x^{\frac{2}{3}}(x-15)$ on the interval $[-8,1]$.

SCORE: _____ / 7 PTS

$= x^{\frac{5}{3}} - 15x^{\frac{2}{3}}$

$f'(x) = \frac{5}{3}x^{\frac{2}{3}} - 10x^{-\frac{1}{3}}$ $\underline{\text{DNE @ } x=0}$ 1

1 $\underline{\frac{5}{3}x^{-\frac{1}{3}}(x-6)=0}$ $\underline{\text{@ } x=6}$ $\notin [-8,1]$

1

x	f(x)
-8	-92
0	0
1	-14

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

ABSOLUTE $\underline{\text{MAX @ } x=0}$ 1

$\underline{\text{MIN} \quad x=-8}$ 1

+ $\frac{1}{2}$ FOR NO OTHER x, f(x) VALUES

Evaluate the following limits. **SCORE: _____ / 10 PTS**

<u>Your answer should be a number, ∞, –∞ or DNE (only if the first three answers do not apply).</u>

[a] $\lim_{x\to 0^+}(2-e^x)^{\frac{1}{x}}$ $\quad 1^\infty$

$= \lim_{x\to 0^+}\left[e^{\ln(2-e^x)}\right]^{\frac{1}{x}}$

$= \boxed{e^{\lim_{x\to 0^+}\frac{\ln(2-e^x)}{x}}}$ $\quad 1\frac{1}{2}$

$= \boxed{e^{-\frac{1}{2}}}$ $\quad 1\frac{1}{2}$

$\lim_{x\to 0^+}\frac{\ln(2-e^x)}{x}$ $\quad \frac{0}{0}$

$= \boxed{\lim_{x\to 0^+}\frac{\frac{1}{2-e^x}\cdot -e^x}{1}}$ $\quad 1$

$= \frac{1}{2-1}\cdot -1$

$= \boxed{-\frac{1}{2}}$ $\quad \frac{1}{2}$

[b] $\lim_{x\to 0}\frac{x(1-\cos x)}{x-\sin x}$ $\quad \frac{0}{0}$

$1\frac{1}{2}$ $\boxed{= \lim_{x\to 0}\frac{1-\cos x + x\sin x}{1-\cos x}}$ $\quad \frac{0}{0}$

$= \lim_{x\to 0}\frac{\sin x + \sin x + x\cos x}{\sin x}$

$1\frac{1}{2}$ $\boxed{= \lim_{x\to 0}\frac{2\sin x + x\cos x}{\sin x}}$ $\quad \frac{0}{0}$

$1\frac{1}{2}$ $\boxed{= \lim_{x\to 0}\frac{2\cos x + \cos x - x\sin x}{\cos x}}$

$= \boxed{3}$

1

Consider the following three cases: GRADED BY ME SCORE: _____ / 8 PTS

Case 1: $f(x) = \cos\frac{1}{x}$ on the interval $[-\frac{\pi}{4}, \frac{\pi}{4}]$ $\longleftarrow$ NOT DEFINED/CONTINUOUS @ $x=0$

Case 2: $g(x) = x^{\frac{2}{5}}$ on the interval $[-32, 32]$ $\longleftarrow$ $g'(x) = \frac{2}{5}x^{-\frac{3}{5}}$ DNE @ $x=0$

Case 3: $h(x) = x + \frac{1}{x}$ on the interval $[-4, -1]$

[a] In which cases does the Extreme Value Theorem apply ? If the Extreme Value Theorem does **not** apply in any case, write "N/A".
For the cases in which the Extreme Value Theorem applies, list all the conditions of the Extreme Value Theorem which are satisfied.

CASES 2, 3: FUNCTIONS ARE CONTINUOUS ON CLOSED + BOUNDED INTERVALS

[b] In which cases does Rolle's Theorem apply ? If Rolle's Theorem does **not** apply in any case, write "N/A".
For the cases in which Rolle's Theorem applies, list all the conditions of Rolle's Theorem which are satisfied.

N/A

[c] In which case does the Mean Value Theorem apply ?
List all the conditions of the Mean Value Theorem which are satisfied in that case, and find the value of c guaranteed by the theorem.

CASE 3: h IS CONT ON $[-4,-1]$ AND DIFF ON $(-4,-1)$

$$h'(c) = 1 - \frac{1}{c^2} = \frac{(-1 + -1) - (-4 + -\frac{1}{4})}{-1 - -4} = \frac{-2 + \frac{17}{4}}{3} = \frac{3}{4}$$

$$\frac{1}{4} = \frac{1}{c^2}$$

$c = \pm 2$ $\qquad$ $c = -2 \in (-4,-1)$

Using complete sentences and proper mathematical notation, state the **formal definition** of "local minimum". **SCORE: _____ / 2 PTS**

f HAS A LOCAL MINIMUM AT C IF $f(c) \leq f(x)$ FOR ALL X NEAR C

GRADED BY ME

Using complete sentences and proper mathematical notation, state Rolle's Theorem. **SCORE: _____ / 3 PTS**

IF f IS CONTINUOUS ON $[a,b]$ AND f IS DIFFERENTIABLE ON (a,b)

AND $f(a)=f(b)$

THEN $f'(c)=0$ FOR SOME $c \in (a,b)$

GRADED BY ME